
EXERCISE SHEET : OTHER APPLICATIONS OF INTEGRALS

1. Consider the function $f(x) = \frac{x}{6}$.
 - i) Find the average of $f(x)$ in the interval $[0, 6]$.
 - ii) Find $c \in [0, 6]$ such that $f(c)$ equals the average.
2. Let $f(x) = 2x^2 - 1$. Given that $\int_0^3 f(x)dx = 15$, find a point $c \in [0, 3]$ such that $f(x)$ equals the average of $f(x)$.
3. Consider a parabola $f(x) = -x^2 + 2x + 1$. In which interval $[a, b]$ is the integral $\int_a^b f(x)dx$ as large as possible?

4. Consider the function

$$f(x) = \begin{cases} 1 - |x - 1| & 0 \leq x < 2 \\ -\sqrt{4 - (x - 4)^2} & 2 \leq x < 6 \\ 3 - |x - 9| & 6 \leq x < 12 \end{cases}$$

Find the average of $f(x)$.

5. Compute $\int_1^2 \ln(x)dx$ using a Riemann sum :
 - i) Divide the interval $[1, 2]$ in n subintervals. How long is each interval?
 - ii) Evaluate the function $f(x) = \ln(x)$ in the right endpoint of each subinterval.
 - iii) Write the Riemann sum.
 - iv) Deduce the value of the integral assuming that the limit when $n \rightarrow +\infty$ of the Riemann sum is $\log(4) - 1$.